Perturbative treatment of the multichannel interacting resonant-level model in steady-state nonequilibrium

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We consider the steady-state nonequilibrium physics of the multichannel interacting resonant-level model in the weak coupling regime. By using the scattering state method we show in agreement with the rate equations that the negative differential conductance at large enough voltages is due to the renormalization of the hopping amplitude and neither the orthogonality catastrophe nor the voltage dependence of the density of states at the resonant-level. The moderate role of the voltage dependence of the density of states on the dot is also discussed.

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I. INTRODUCTION

Since the experimental realization of quantum dots the electronic transport in mesoscopic systems has attracted fastly growing interest.[1] In these systems the short-range Coulomb interaction plays an important role and may result in strong correlations between the electrons on the dot and the electronic leads.

In the Brief Report, we consider a single level quantum dot (QD) with spinless electrons thus only the interaction between the dot and the leads is taken into account. The dot is attached to two lead electrodes via direct tunneling. In addition to that the short-range Coulomb interaction is considered which is acting between the dot electron and the electrons on the neighboring sites of the leads as sketched in Fig. 1.

It is interesting to note that a somewhat related setup has been studied in earlier works dealing with the problem of dephasing. Levinson[2] and Aleiner *et al.*[3] have studied the problem of dephasing of an electron in a quantum dot due to the presence of a capacitively coupled point contact (PC). That setup can serve as a "which path" detector in an interference experiment if the QD is embedded into one of the arms of an Aharonov-Bohm interferometer. That setup shares some common feature with the situation discussed in the present manuscript but the problem of dephasing of electrons is beyond the scope of the present paper.

II. MODEL AND METHOD

Focusing now on the setup sketched in Fig. 1 we can assume—without loss of generality—that the hopping is restricted between the dot and leads 1 and 2 while the Coulomb interaction acts to all of the N leads. The model is described by the Hamiltonian

$$H = \sum_{k,i=1\ldots N} \varepsilon_{ki} c_{ki}^{\dagger} c_{ki} + \varepsilon_d d^{\dagger} d + t_0 \sum_{k,i=1,2} d^{\dagger} c_{ki} + c_{ki}^{\dagger} d + \frac{U}{2}\left(d^{\dagger} d - \frac{1}{2}\right) \sum_{k,i=1\ldots N} \left(c_{ki}^{\dagger} c_{ki} - \frac{1}{2}\right), \quad (1)$$

where $c_{ki}^{\dagger}$ (c_{ki}) creates (annihilates) an electron in the lead i with momentum k, d stands for the dot electron annihilation operator. t_0 is the hybridization amplitude between the dot and the leads 1 and 2, while U is the strength of the Coulomb interaction acting between the dot and all the leads. The substraction of 1/2 in the last term is due to practical reasons: in this form the Hamiltonian obeys electron-hole symmetry if the local level is tuned to $\varepsilon_d = 0$. In the following we restrict our investigations to the electron-hole symmetric case therefore we do not discuss gate voltage (i.e., ε_d) dependence of the transport through the QD. Concerning the role of the level position we refer the reader to two recent publications Refs. 4 and 5.

The equilibrium physics of the model has been intensively studied in the recent decades.[6–10] It is worth to mention that a related model has been studied very recently in the context of fractional quantum Hall systems,[11] but that parafermionic version of the interacting resonant-level model is certainly beyond the scope of the present paper. In the context of the fermionic model the steady-state nonequilibrium situation—beyond perturbation theory—[10,12]has been studied by exact and numerically exact approaches including Bethe ansatz,[13]

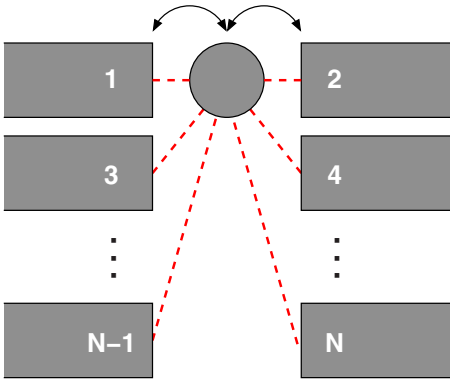

FIG. 1. (Color online) Possible realization of the multichannel interacting resonant-level model. A single level quantum dot is attached to several lead electrodes. Hopping between the dot level and the neighboring sites of the leads $N=1,2$ are present in addition to the short range Coulomb interaction acting between the dot electron and the electrons on the neighboring sites of all N electrodes. The voltage is applied between leads 1 and 2.

conformal field theory and time-dependent density matrix renormalization group.[14–16]

By applying these methods the I-V characteristics of the model has been calculated where the external voltage applied between the conduction electron leads labeled by 1 and 2 (left/right). Even if the exact methods give correct answers it is hard to get insight into the physical mechanisms and the different competing processes. The perturbative methods can provide help in the better understanding.[10,12] Recently, at high bias a rather surprising phenomenon of the negative differential conductance has been found by exact methods.[14,17] The aim of the present paper is to provide hints by perturbative methods how that negative differential conductance is established. To that end we study the many channel version of the interacting resonant-level model. Even though the negative differential conductance shows up in the $N=2$ channel case already for arbitrarily small repulsive interaction $U>0$ it is of some advantage to treat the case of larger N. Perhaps the most important advantage is that the nonmonotonic dependence of the current on the interaction strength at intermediate voltages[13] can only be described within the framework of perturbation theory if $N \gg 1$.

Following our earlier work the scattering state method combined with time dependent perturbation theory will be applied. That method is suitable for out of equilibrium situations as in the method any initial state can be considered. The renormalization is taken into account by a generalization of Anderson's poor man's scaling including the corrections in next-to-leading logarithmic order and the renormalization of the initial and final states.[10,18]

III. RESULTS

In the spinless model two competing phenomenon occur:

(i) If the impurity level is occupied then the repulsive Coulomb interaction pushes away the electrons in the leads from the vicinity of the dot therefore creating more unoccupied states which helps the dot electron to tunnel to one of the leads. In this way the Coulomb interaction enhances the hopping rate.

(ii) When the occupation of the dot is changed then it has to be followed by the rearrangement of the electrons in all of the leads due to the Coulomb interaction. This rearrangement is similar to Anderson's orthogonality catastrophe and it takes a long time to be completed. This mechanism essentially reduces the hopping rate.

These two processes are competing in case of repulsive Coulomb interaction U. In perturbation theory the former one appears already in first order of U, while the latter one in second order only. Therefore a crossover is expected by increasing U.[10] In case of two leads that crossover occurs in the medium/strong coupling regime which is already outside of the validity region of the perturbative treatment. The crossover can be pushed down to the weak coupling regime by increasing the effect of the Coulomb repulsion by increasing the number of screening channels (leads), e.g., to $N=10$.

As in Ref. 10 the self-energy and the vertex corrections are given by the skeleton diagrams in Fig. 2. The solid lines represent the conduction electron propagators in different channels and the dashed line the electron on the dot. Diagrams (a) and (b) are the bare vertices, (c) is the self-energy, (d) the vertex correction to the hopping in first order, and (e) in second order.

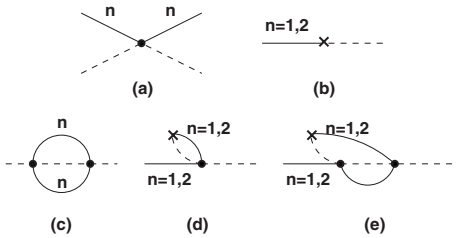

FIG. 2. The bare Coulomb vertex (a) and hybridization vertex (b). The solid lines represent the conduction electron propagators in different channels and the dashed line the electron on the dot. Diagram (c) is the self energy, (d) the vertex correction to the hopping in first order and (e) in second order. The flow of time agrees with the direction of the dotted line.

The self-energy and vertex corrections show different features concerning the role of the applied voltage. In the self-energy diagrams the occupation of the dot is not changed consequently the value of the applied voltage is not relevant as only electron-hole pairs are created at one of the Fermi surfaces. In contrast, in the process responsible for the vertex correction the occupation of the dot is changed by a hopping event thus the voltage V is important and plays the role of the infrared cutoff.

The vertex correction in first order of U [see Fig. 2(d)] is similar to a Hartree diagram: it is independent of the incoming energy ω but depends on the voltage as $Ut \log[D/(\mathrm{eV}+\omega_c)]$ where ω_c is the infrared cutoff arising from the width of the resonance.[19] It is not necessarily the case for the correction of higher order. E.g., the correction depicted in Fig. 2(e) gives

$$U^2 t_0 \log\frac{D}{\mathrm{eV}} \log\frac{D}{|\omega|+\omega_c}, \tag{2}$$

for $\omega \gg \mathrm{eV}$. That may lead to difficulties to determine the renormalization of the vertex for large energies. However, in the transport properties only the electrons in the energy window $-\mathrm{eV}/2<\omega<\mathrm{eV}/2$ play a role thus the appearance of such explicit energy dependence is out of the focus of the present paper.

The scaling equations provide the expression[19]

$$t(\omega) = t_0 \left(\frac{\mathrm{eV}/2}{D}\right)^{-\varrho_0 U} \left(\frac{|\omega|+\Gamma_0}{D}\right)^{N(\varrho_0 U)^2/2}, \tag{3}$$

where ϱ_0 is the conduction electron density of states and it is assumed that the width of the resonance $\Gamma_0 = 2\pi\varrho_0 t_0^2$ can be neglected in comparison with the applied voltage eV. The result shown in Eq. (3) is very similar to that obtained by Matveev and Larkin.[9] There is, however, a crucial difference between their result and ours: in their formula the applied voltage appears as a low energy cutoff both in the vertex and the self energy corrections in the same fashion which is in contrast with our findings.

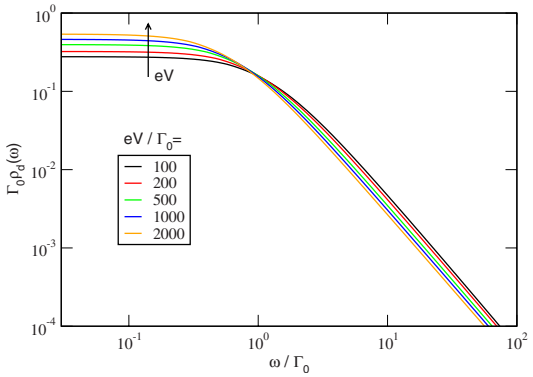

FIG. 3. (Color online) The normalized impurity density of states for $N=10$ channels at $\varrho_0 U=0.1$ and different values of the bias voltage. $\Gamma_0=2\pi\varrho_0 t_0^2$.

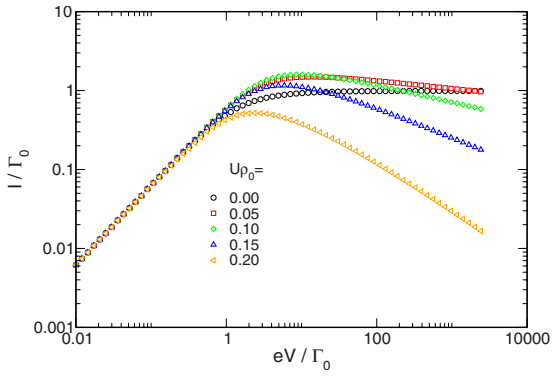

FIG. 4. (Color online) *I-V* characteristics with the clear evidence of the nonmonotonic behavior for the $N=10$ channel interacting resonant-level model for different values of the Coulomb repulsion.

As in Ref. 10 the rate equations are used to determine the steady-state current between the dot and one of the leads due to the voltage eV applied on the leads [see Eqs. (28)–(31) of Ref. 10]. In the scattering state method it is more feasible to calculate the scattering amplitude of an electron going from one of the leads to the other one. In the framework of the time ordered diagrams the first and last hopping must be picked up and the propagation of the electron on the dot can be described by the inverse life time $\Gamma(\omega)/2$ which contains the contribution (to all orders) of the hopping to the leads. In the initial state the impurity can either be occupied or unoccupied but the results are very similar. Thus that amplitude is

$$t(\omega)\frac{1}{\omega+i\Gamma(\omega)/2}t(\omega), \tag{4}$$

and in this way the current is

$$I(\mathrm{eV})=2\pi\varrho_0 e\int_{-\mathrm{eV}/2}^{\mathrm{eV}/2}t^4(\omega)\left|\frac{1}{\omega+i\Gamma(\omega)/2}\right|^2 d\omega. \tag{5}$$

where the current flows in the energy window $-\mathrm{eV}/2<\omega<\mathrm{eV}/2$ if the voltage bias applied symmetrically. $t(\omega)$ appearing in (5) is determined self consistently, i.e., its RG flow is terminated either by ω, the voltage eV or by $\Gamma(\omega)$ itself according to Eq. (3). Using the relation $2\pi\varrho_0 t^2(\omega)=\Gamma(\omega)$ and

$$\varrho_d(\omega)=\frac{1}{\pi}\frac{\Gamma(\omega)/2}{\omega^2+[\Gamma(\omega)/2]^2},$$

we obtain

$$I(\mathrm{eV})=2\pi e\int_{-\mathrm{eV}/2}^{\mathrm{eV}/2}t^2(\omega)\varrho_d(\omega). \tag{6}$$

Comparing Eqs. (5) and (6) it is interesting to notice that a factor t^2 is now incorporated by the density of states ϱ_d. The result given by Eq. (6) is just the current going to the dot from, e.g., the left electrode. That holds only in the special case with L/R symmetry ($t_L=t_R$). It is worth mentioning that in this special case the results obtained by using rate equations and the scattering state method coincide.

Considering the voltage dependence of the current two factors play role. According to Eq. (3) the voltage dependence of the hopping rate $t(\omega)$ is determined by the vertex correction which is reduced if the voltage is increased. On the other hand the renormalized density of states ϱ_d is increasing with increasing voltage in the relevant energy window (see Fig. 3). As the final result shows current reduction at large enough voltages, therefore the dominating process is (i) (the vertex renormalization) and the orthogonality catastrophe [process (ii)] just modifies its amplitude in a voltage independent manner (see Fig. 4).

IV. CONCLUSION

In conclusion, we have shown that remarkable physics of negative differential conductance in the interacting resonant-level model previously studied by exact methods can be understood on the ground of perturbation theory. Such a treatment of the problem—though it is restricted to the weak coupling regime of the model—permits us to gain insight into the mechanisms leading to negative differential conductance at large voltages. Our conclusion is that the underlying mechanism is the vertex renormalization—discussed earlier as phenomenon (i)—which has already been analyzed in the equilibrium case but now it is modified by the presence of the applied voltage. We have shown that the negative differential conductance at large enough voltages is due to the renormalization of the hopping amplitude and neither due to the orthogonality catastrophe nor the voltage dependence of the density of states at the resonant-level. To the best of our knowledge the moderate role of the voltage dependence of density of states on the dot is discussed first time.

As the interacting resonant-level model has become a benchmark model for different theoretical methods dealing with nonequilibrium physics of quantum impurity problems, it would be nice to check whether the more precise RG methods[20–24] provide conclusion similar to ours.

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